Journal of Engineering Physics and Thermophysics, Vol. 70, No. 3, 1997

CALCULATION OF THE EMERGENCY REGIMES OF CRYOSTATS

Yu. L. Buyanov **UDC 621.642:629.786**

Analytical expressions are obtained that allow one to evaluate the dependence between the time of pressure growth in cryostats containing cryogenic fluids and the thermal energy entering the cryostat.

Introduction. Emergency conditions in cryostats develop when a cryogenic agent is supplied with appreciable heat fluxes greatly exceeding those specified by design in heat bridges and insulation. Such situations are typical on loss of vacuum in the insulation cavity of a cryostat, during operation of any device with excessive power in a cryogenic fluid, or, which is particularly characteristic, in transition to the normal state of the superconducting winding of a magnet immersed in liquid helium.

The inflow of large amounts of heat to a cryogenic agent causes its intense evaporation and, in the case of restricted outflow of vapor or of a vapor–liquid mixture through the gas release line or protective fittings, leads to an inadmissible growth of pressure in the cryostat, which may cause its damage. To provide safe conditions of operation, it is necessary to know the time during which in the cryostat when there is an active heat source of known constant or variable power, an acceptable level of pressure not exceeding, for example, the operating pressure of protective devices of certain throughput capacity is ensured. Another problem amounts to determining the admissible power of heat release in the cryostat or finding the throughout capacity of its protective fittings and gas release line.

Methods for calculating pressure in a cryostat in emergency conditions have been considered by a number of authors [1, 2]. The solutions in these works were found by using a numerical method [1] or nomograms [2]; they do not provide a clear representation of the dependence between the physical parameters characterizing the processes in a cryostat. In the present work an attempt is made to use an analytical method for calculating emergency regimes of cryostats.

Statement of Problem and Calculation Technique. On addition of a large amount of heat to vessels with cryogenic fluids leading to a rapid change in pressure, the processes occurring in these vessels are made complicated by the formation of nonequilibrium states of the two-phase liquid–vapor medium and its structural nonuniformity. For describing such complex processes, many authors, while determining the dependences between thermal parameters and caloric functions, use relations of quasistatic equilibrium thermodynamics. In the present work, a similar simplified model is used within the framework of which an assumption about the preservation of the mutual equilibrium of phases allows one to make an approximate calculation of the relationship between the energy entering a cryostat and the pressure in it. Also, the following assumptions were made: the pressure in the cryostat does not exceed the critical pressure of the cryogenic fluid, the pressure and temperature in the entire volume of the cryostat bath are identical, and the ambient pressure is constant.

A $T-S$ diagram of the thermodynamic process occurring in the cryostat is presented in Fig. 1. In a closed vessel, or when the rate of evaporation of the cryogenic agent greatly exceeds the throughput capacity of the drain lines, the thermodynamic process (0-1) will follow an isochore that depends on the degree of vessel filling $\psi = G_{\mathrm{cr}}(\tau)/G^{*}$. Calculation of closed vessels (without drainage) is described in [3, 4]. On attainment of pressure p_1, the cryogenic agent is discharged through a branch pipe and protective fittings, i.e., a valve or blowout diaphragm; in this case the process proceeds with a certain increase in pressure (1-2′) or at a constant pressure (1-2). The process is characterized by a continuous change in the specific volume of the cryogenic agent that depends on the quantity ψ.

Joint Institute for High Temperatures, Russian Academy of Sciences, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 70, No. 3, pp. 413-419, May-June, 1997. Original article submitted June 16, 1995.

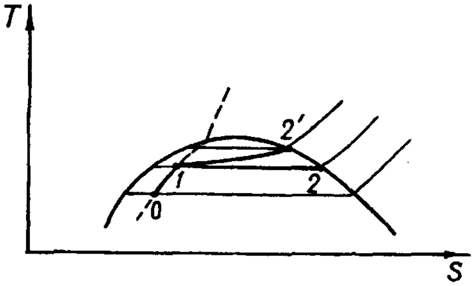

Fig. 1. $T-S$ diagram of process of heat supply to cryostat (solid curves, constant pressure; dashed curve, constant specific volume): 0) beginning of process; 1) opening of protective fittings; 2 or 2') end of process (if cryogenic agent escapes from cryostat as saturated vapor).

Let us write in differential form the energy balance of the thermodynamic system considered

$$dW + dW_{\text{ins}} = dU + dI, \tag{1}$$

where dW is the energy released in the cryostat, for example, in transition of a superconducting magnet to the normal state or as a result of the action of any heat source for a definite period of time; dW_{ins} is the thermal energy associated with heat inflow through the thermal insulation of the cryostat. Since $W >> W_{\text{ins}}$, this portion of energy will be neglected in further analysis.

Thermal energy dW is spent in changing the internal energy of the cryogenic agent dU, i.e., in increasing the pressure in the cryostat, and its portion dI escapes with the vapor released. Expanding the components of the energy balance, we write

$$dU = d\,(G_{\text{cr}}u) = dG_{\text{cr}}u + G_{\text{cr}}du\,,$$

$$dI = i_{\text{esc}}dG_{\text{esc}}\,,$$

where

$$G_{\text{cr}} = G_0 - \int_0^{\tau} m d\tau\,; \quad G_{\text{esc}} = \int_0^{\tau} m d\tau\,.$$

Substituting dU and dI into Eq. (1) and dividing all the terms of the energy balance equation by G_{cr}, we obtain

$$\frac{dW}{G_{\text{cr}}} = \frac{dG_{\text{cr}}}{G_{\text{cr}}}u + du + \frac{i_{\text{esc}}dG_{\text{esc}}}{G_{\text{cr}}}. \tag{2}$$

We will determine now the terms of Eq. (2). We will find the ratio $dG_{\text{cr}}/G_{\text{cr}}$ from the equation of state $pV = G_{\text{cr}}zRT$, having differentiated the right- and left-hand sides of the equation at constant values of z and R:

$$\frac{dG_{\text{cr}}}{G_{\text{cr}}} = \frac{dp}{p} - \frac{dT}{T}.$$

Substituting into the equation that defines the specific internal energy of the cryogenic agent $u = i - pv$, the expression for the enthalpy of moist vapor

$$i = i' + (v - v')\,T\frac{dp}{dT},$$

we obtain

$$u = i' + (v - v')\, T \frac{dp}{dT} - pv. \tag{3}$$

From the Clapeyron–Clausius equation applied to vapor–liquid mixtures we find

$$\frac{dp}{dT} = \frac{r}{T\,(v'' - v')}. \tag{4}$$

Substitution of Eq. (4) into Eq. (3) yields

$$u = i' + \frac{v - v'}{v'' - v'}\, r - pv.$$

In engineering calculations we may assume that $i' \approx c_v' T$ [5]. Since $pv = zRT$, we finally obtain

$$u = c_v' T + \frac{v - v'}{v'' - v'}\, r - zRT.$$

The total differential of the change in the specific internal energy can be written in the form

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv,$$

where $(\partial u/\partial v)_v = c_v$ and $(\partial u/\partial v)_T dv = pT/z(\partial z/\partial T)_v$. At $z = \text{const}$, we have $du = c_v dT$.

Substituting into Eq. (2) expressions that define dG_{cr}/G_{cr}, u, and du, we obtain

$$\frac{dW}{G_{cr}} = (c_v - c_v' + zR)\, dT + \frac{dp}{p}\, T\,(c_v' - zR) +$$

$$+ \frac{dp}{p}\, \frac{v - v'}{v'' - v'}\, r - \frac{dT}{T}\, \frac{v - v'}{v'' - v'}\, r + \frac{i_{esc} dG_{esc}}{G_{cr}}. \tag{5}$$

For the convenience of the subsequent integration we transform in the equation the term $(dp/p)T(c_v' - zR)$, having substituted into it the quantities $dp = (dT/T)(r/(v'' - v')$ and $p = zRT/v$. Then

$$\frac{dp}{p}\, T\,(c_v' - zR) = \frac{r}{zR}\, \frac{v}{v'' - v'}\, \frac{dT}{T}.$$

After this, Eq. (5) takes on the form

$$\frac{dW}{G_{cr}} = a_1 dT + a_2 \frac{dT}{T} + a_3 \frac{dp}{p} - a_3 \frac{dT}{T} + \frac{i_{esc} dG_{esc}}{G_{cr}}, \tag{6}$$

where

$$a_1 = c_v - c_v' + zR\,; \quad a_2 = (c_v - zR)\, \frac{r}{zR}\, \frac{v}{v'' - v'}\,; \quad a_3 = \frac{v - v'}{v'' - v'}\, r.$$

We will assume that the physical parameters are determined at mean values of pressure $\bar{p} = (p_1 + p_2)/2$ or temperature $\bar{T} = (T_1 + T_2)/2$, where p_1 and T_1 are the pressure and temperature in the cryostat at time $\tau = 0$, and p_2 and T_2, at time τ. Taking account of the fact that $zR = \bar{p}v/\bar{T}$, we obtain

$$a_1 = c_v - c_v' + zR\,; \quad a_2 = (c_v - zR)\, \frac{r}{zR}\, \frac{v}{v'' - v'}\,; \quad a_3 = \frac{v - v'}{v'' - v'}\, r.$$

The specific volume of a two-phase mixture entering into the coefficients a_1, a_2, and a_3 is determined using an equation from [3]:

$$v = \frac{v' v''}{\psi v'' + (1 - \psi) v'},$$

and the specific heat is determined by means of an equation from [5]:

$$c_v = c_v' + \frac{v - v'}{v'' - v'} (c_v'' - c_v').$$

We integrate Eq. (6) within the limits of the change of the pressure and temperature in the cryostat for time τ:

$$\int_0^\tau \frac{dW}{G_{cr}} - \int_0^\tau \frac{i_{esc} dG_{esc}}{G_{cr}} = a_1 \int_{T_1}^{T_2} dT + (a_2 - a_3) \int_{T_1}^{T_2} \frac{dT}{T} + a_3 \int_{p_1}^{p_2} \frac{dp}{p}. \quad (7)$$

The right-hand side of Eq. (7) represents a change in the internal energy related to a unit mass of cryogenic agent. Having integrated it, we write Eq. (7) in the form

$$\int_0^\tau \frac{dW}{G_{cr}} - \int_0^\tau \frac{i_{esc} dG_{esc}}{G_{cr}} = A, \quad (8)$$

where $A = a_1(T_2 - T_1) + (a_2 - a_3) \ln (T_2/T_1) + a_3 \ln (p_2/p_1)$.

If we assume that the flow rate of the cryogenic agent that escapes from the cryostat is constant in the range of pressures p_1 and p_2 (where p_1 is the opening pressure of the protective device and p_2 is the prescribed maximal calculated pressure established in the cryostat, for example, within the limits of the adjustment of the protective device $\Delta p = p_2 - p_1$), then $G_{cr} = G_0 - m\tau$ and $dG_{esc} = md\tau$.

As a result, Eq. (8) takes on the form

$$\int_0^\tau \frac{dW}{G_0 - m\tau} - i_{esc} \ln \frac{G_0}{G_0 - m\tau} = A. \quad (9)$$

The flow rate of the gas escaping from the cryostat is determined from the equation $m = fc$, where f is the cross-sectional area of the flow portion of the protective fittings and gas release branch pipes. The quantity c, which is the ratio of the flow velocity of the cryogenic agent escaping from the cryostat to the specific volume, is determined differently using the well-known equations, depending on the assumptions made regarding the state of the cryogenic agent leaving the cryostat. For example, on the assumption of the adiabatic escape of vapor, $c = (2\Delta i)^{1/2}/v''$, where Δi is the difference in the enthalpies of the cryogenic agent at the entrance and exit from a branch pipe.

Two cases of solving Eq. (9) are possible: in the first case heat is supplied to the liquid cryogenic agent from a heat source of constant power Q, and then $dW = Qd\tau$, whereas in the second case, a body heated to a certain temperature cools off in the vessel of the cryostat transmitting its thermal energy to the cryogenic agent, i.e., $dW = Q(\tau)d\tau$.

Action of a Heat Source of Constant Power. At a constant power of the object of heat generation, integration of Eq. (9) gives

$$\left(\frac{Q}{m} - i_{esc}\right) \ln \frac{G_0}{G_0 - m\tau} = A. \quad (10)$$

We can calculate any parameter from Eq. (10). For example, one has to find the power of a heat source in a cryostat at which the pressure in it increased from p_1 to p_2 in a certain time τ. In this case the needed power of heat generation is defined by the relation

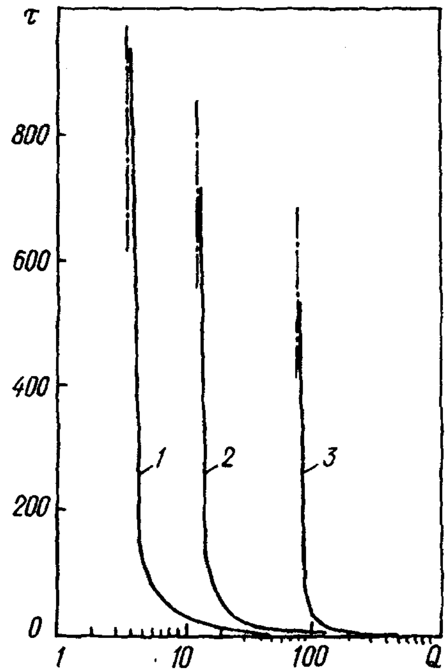

Fig. 2. Time of pressure growth in helium cryostat depending on power of heat generation (the initial pressure is 0.15 MPa, the final pressure is 0.17 MPa). The parameter is the diameter of the hole for releasing vapor: 1) 10 mm, 2) 20, 3) 50. The dashed-dotted lines correspond to $Q = mi_{esc}$ for holes of different diameters. τ, sec; Q, kW.

$$Q = m\left(\frac{A}{\ln \dfrac{G_0}{G_0 - m\tau}} + i_{esc}\right). \tag{11}$$

When a calculation is performed using Eq. (11), we determine the value of A from the prescribed values of p_1 and p_2 and establish the dependence $Q = f(\tau)$. We note that when the pressure in the cryostat does not rise under the action of a heat source, i.e., thermal energy is not spent in changing the internal energy ($A = 0$), this occurs at $Q = mi_{esc}$. If one needs to determine the time of the change in pressure from p_1 to p_2 at the prescribed power of heat generation, then from Eq. (10) we obtain

$$\tau = \frac{G_0}{m}\left\{1 - \exp\left[A\Big/\left(i_{esc} - \frac{Q}{m}\right)\right]\right\}. \tag{12}$$

Since A depends on the specific volume of the cryogenic agent, and this volume, in turn, depends on the vessel filling degree ψ, which is determined by the duration of the process, then the calculation of τ by Eq. (12) is made by the method of successive approximations.

As an example, Fig. 2 presents graphs of the dependence $\tau = f(Q)$ that show the time in which the pressure will rise in a cryostat filled with liquid helium with an active heat source of a certain power. The graphs were plotted for different values of the diameter of the hole through which helium escapes from the cryostat with time. In this example $p_1 = 0.15$ MPa, $p_2 = 0.17$ MPa. $G_0 = 100$ kg, and $V = 1$ m^3. Helium leaves the cryostat in the form of saturated vapor, i.e., $i_{esc} = i''$. As follows from the graphs, $\tau \to \infty$ when $Q \to mi_{esc}$, which determines the minimum power at which there is no rise in pressure.

Power of a Heat Source as a Function of Time. We will consider this case using as an example the transition of a superconducting magnet to the normal state. In transition, a portion of the energy stored in the magnet is spent in heating the winding. Leaving outside the brackets the rapidity of this heating, determined by the speed of propagation of the normal zone in the conductor, the time of current decay in the winding, the design features of the magnet, and by the conditions of heat exchange in the liquid helium, we assume that at the initial time instant ($\tau = 0$) the entire volume of the idle winding is at temperature T_0, and subsequently it cools off uniformly in the medium of the cryogenic fluid.

The main problem is reduced to the determination of the integral $\int_0^\tau (dW/(G_0 - m\tau))$ in Eq. (9). The quantity of heat given up by the body (magnet) during cooling can be found from the equation

$$W = Q_0 \left[1 - \bar{\theta}(\tau)\right], \tag{13}$$

where $\bar{\theta}(\tau) = (\theta - T)/(\theta_0 - T)$ is the dimensionless temperature of the body, which is the mean over the volume or along any of the coordinates (for a one-dimensional problem) at any time instant; θ_0 is the quantity of heat transferred to helium during body cooling. The upper limit of the value Q_0 is equal to the energy stored in the magnet W_0.

We shall assume that the body temperature changes according to an equation of the following form [6]:

$$\bar{\theta} = P \exp(-\mu_1^2 \, \mathrm{Fo}), \tag{14}$$

where $\mathrm{Fo} = (a\tau)/l^2$; $\mathrm{Bi} = (\alpha l)/\lambda$.

The quantity P, being a function of the Biot number, is determined, depending on the shape and dimensions of the body, from equations or tables familiar in heat conduction.

Differentiating Eq. (13) after substitution of Eq. (14) into it, we obtain

$$dW = \frac{Q_0 P \mu_1^2 a}{l^2} \exp(-\mu_1^2 \, \mathrm{Fo}) \, d\tau . \tag{15}$$

Relation (15) makes it possible to determine the value of the integral in Eq. (9):

$$\int_0^\tau \frac{dW}{G_0 - m\tau} = \frac{Q_0 P \mu_1^2 a}{l^2} \int_0^\tau \frac{\exp(-\mu_1^2 \, \mathrm{Fo})}{G_0 - m\tau} \approx K \left(\ln \frac{G_0}{G_0 - m\tau} + \mu_1^2 \, \mathrm{Fo} \right), \tag{16}$$

where

$$K = \frac{Q_0 P \mu_1^2 a}{m l^2} \exp\left[(-\mu_1^2 a G_0)/(m l^2)\right].$$

The final form of Eq. (9) is

$$(K - i_{\mathrm{esc}}) \ln \frac{G_0}{G_0 - m\tau} + \frac{Ka}{l^2} \tau = A . \tag{17}$$

Relation (17) allows one, just as in the case of a constant power, to determine the time during which the pressure changes in a cryostat in which a body cools off after rapid heating, transmitting its thermal energy to the cryogenic agent.

Using the relations obtained for the two cases considered, it is convenient to analyze different regimes and establish the connection between the emergency heat generation and the cross-sectional area of branch pipes needed for vapor release. We note that a sufficiently accurate calculation of the needed quantities, for example, of the time of pressure rise for the case of heat power depending on time, involves rather accurate determination of the Biot number and, consequently, of the heat exchange coefficient, whose numerical value very closely approximates the objective data, as well as correct averaging of the thermal conductivity and thermal diffusivity coefficients, which change in a rather wide range of temperatures: from the initial temperature of the heated body to the temperature of the liquid cryogenic agent. Therefore, there is a reliable way of assigning convenient determining temperatures by introducing into Eq. (17) correction coefficients found from comparison of the predicted and experimental data for nearly similar cases. Another possibility is calculation for small changes Δp, from which one must determine the values of $\Delta \tau$ corresponding to these changes.

It is also possible to carry out calculations by simpler equations (10)-(12), having averaged the heat source power. In particular, the utility of such an approach was revealed by a verifying calculation of the power that was scattered by the guard resistors of the superconducting coil described in [7]. According to the operational conditions, all the energy stored in the coil is liberated inside the cryostat. In preliminary tests of this magnetic system, the pressure in a cryostat containing 330 liters of liquid helium increased from 0.17 to 0.22 MPa 5 sec after opening of the protective valves, because of their small flow cross-sections (the nominal diameters of the valves were 80 and 25 mm. The entire helium in the liquid state was expelled from the cryostat in 13 sec.

Proceeding from the induction and resistance values of the coil, the mean power of heat generation was equal to 0.095 MW. Calculation by Eq. (11) showed that for a mean velocity of liquid helium discharge of 3.17 kg/sec and enthalpy $i_{esc} = 14.5 \cdot 10^3$ J/kg, corresponding to mean pressure $\bar{p} = 0.195$ MPa, the power of heat generation must be equal to 0.07 MW, i.e., the error does not exceed 30%.

NOTATION

p, pressure in space with cryogenic agent, Pa; T, temperature in space with cryogenic agent, K; θ_0, initial temperature of cooled body, K; $\bar{\theta}$, mean temperature of body, K; G_0, mass of cryogenic agent at initial time instant, kg; G_{cr}, mass of cryogenic agent in inner space of cryostat at any time instant, kg; G_{esc}, mass of cryogenic agent that escaped from the cryostat, kg; G^*, mass of cryogenic agent in entirely filled inner space of cryostat, kg; ψ, degree of filling of inner space; τ, time, sec; W, thermal energy liberated in cryostat, J; U, internal thermal energy of cryostat, J; I, thermal energy of cryogenic agent escaping from cryostat, J; W_{ins}, thermal energy supplied through insulation, J; W_0, energy stored in superconducting magnet, J; Q_0, quantity of heat transferred during complete cooling of body, J; u, specific internal energy, J/kg; du, change in specific internal energy during time $d\tau$, J/kg; Q, power of heat source, W; i, i_{esc}, enthalpy of cryogenic agent inside cryostat and of cryogenic agent escaping from cryostat, J/kg; S, entropy, kJ/K; m, flow rate of cryogenic agent escaping from the cryostat, kg/sec; r, heat of vaporization of the cryogenic agent, J/kg; R, gas constant, J/(kg·K); z, compressibility factor; c_v, mass isochoric heat capacity, J/(kg·K); V, volume of helium bath, m^3; v, specific volume, m^3/kg; μ_1, root of characteristic equation; l, characteristic dimension of cooled body, m; a, thermal diffusivity , m^2/sec; λ, thermal conductivity of body, W/(m·K); α, coefficient of heat exchange between body and cryogenic agent, W/(m^2·K); Fo, Fourier number; Bi, Biot number; A, J/kg; K, J/kg; P, complexes determined in the text. Subscripts and superscipts: 1 and 2, beginning and end of the process; $'$, liquid phase at saturation line; $''$, vapor phase.

REFERENCES

1. R. P. Krause and E. H. Christensen, in: Proceedings of the 8th Symposium on Engineering Problems of Fusion Research, Vol. 4 (1979), pp. 1765-1768.
2. V. P. Belyakov, S. P. Gorbachev, V. A. Shaposhnikov, et al., in: Proceedings of the All-Union Conference on Engineering Problems of Thermonuclear Reactors in Leningrad [in Russian], Vol. 1 (1977), pp. 257-264.
3. Yu. L. Buyanov, Inzh.-Fiz. Zh., 54, No. 2, 326-327 (1988).
4. V. A. Nevrovskii, Inzh.-Fiz. Zh., 67, Nos. 1-2, 14-19 (1994).
5. M. D. Vaisman, Thermodynamics of Vapor–Liquid Flows [in Russian], Leningrad (1967).
6. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Moscow (1969).
7. G. L. Berezin and Yu. L. Buyanov, Avtomatiz. Sovrem. Tekhnol., No. 4, 14-21 (1994).